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THE POTENCIAL OF ROLEPLAYS USING CONCRETE AND VISUAL REPRESENTATIOS IN TEACHING THE NOTION OF FUNCTION

Introduction

This pilot study delves into the teaching and learning the concept of function, pivotal in algebra education that is typically introduced in later primary school years.

Functions lay the groundwork for mathematical comprehension and that urged authors of this paper to arrange a complex environment that provide students with diverse learning experience which enable them to create symbolic representation of function.

This goal was accomplished by integrating concrete objects, everyday items, visual aids, and role-playing scenarios offering students varied perspectives. Through discussions and role-plays, students gradually developed their understanding of functions. They autonomously constructed concrete and visual representations, eventually translating them into symbolic representations. This approach, coupled with role-plays and tangible objects, proved promising for teaching the concept of functions.

Theoretical background

Constructivism as a learning theory highlights the importance of active student participation in knowledge creation, rather than passive acceptance of information. Glaserfeld's formulated his first principle as: „knowledge is not passively received but actively built up by the cognizing subject".[1] In the process of learning, the learner connects the new information with the knowledge they already have, which they often have to modify in order to successfully accommodate. The key element in this process is that students use their existing, structured knowledge to interpret new information. When there is a contradiction between the new information and the students' internal framework, it can result in a profound transformation of their internal system, a phenomenon referred to as conceptual change in constructivism.[2]

According to Tamás Varga abstraction can only occur from concrete experience,[3] so he recommended to introduce concepts using concrete tools and objects. The literature on teaching and learning theories underscores the significance of

[1] Ernest GLASERFELD: *Constructivism in Education*. The International Encyclopedia of Education (T. Husen & T. N. Postlethwaite, (eds.): Supplement Vol.1. Oxford/New York: Pergamon Press, 2000. 162–163. (Later GRASERFELD 1989)
[2] Iren VIRAG: *Tanuláselméletek és tanítási stratégiák (*Médiainformatikai Kiadványok) Eszterházi Károly Főiskola, Eger, 2013. (Later VIRAG 2013)
[3] Tamas VARGA: *A matematika tanítása,* Tankönykiadó, Bp., 1969.

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representations. Bruner's[4] theory of representation posits three planes – material (enactive), pictorial (iconic), and symbolic – that build upon each other:

1. Material plane (enactive): Knowledge acquisition occurs through concrete material activities, such as using tools.
2. Pictorial plane (iconic): Knowledge is gained through graphic images and imaginary situations, such as Venn diagrams or visual representations of text tasks.
3. Symbolic plane: Knowledge of mathematical symbols and language is developed and utilized. Symbols are divorced from their concrete material form and hold their own meaning.

According to Tall, McGowen and DeMarois,[5] cognitive root is an object phenomenon or procedure which students find easy to understand, and forms the basis of the given conceptual theory. That is, the cognitive "root" (here inafter referred to as the source of the concept) is meaningful cognitive unit of basic knowledge at the beginning of the learning process.

According to function machines serve as versatile tools in mathematics education, aiding children in comprehending and exploring various mathematical concepts in an enjoyable manner. These conceptual tools illustrate the operations of functions, comprising an input stage where values are input, a functional stage where transformation occurs based on predefined rules or algorithms, and an output stage where transformed values are generated.

This model effectively demonstrates the relationship between input and output values governed by a given function. It essentially takes an object within its domain and maps it to a different object, akin to a transformation machine. This conceptualization encapsulates the three key elements of a function: inputs, outputs, and the rule determining their connection.

Taking the above mentioned into account, the function machine is a cognitive root which objectifies the stages of understanding the concept of function (process, object), and enables students to construct the visual representations of the function: table, graph, formula, verbal.

Function machines could be used in role-plays, which help students immerse in roles and imagined situations.

Learning process

After analyzing textbooks, and workbooks, it turned out that they lack emphasis on concrete representations in the process of introducing function concept. Although pictorial representations frequently appear in the textbooks and workbooks, they rarely refer to concrete representations. Acquainted with the relevant scientific and methodological literature, essential statements are

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[4] Jeremy S. BRUNER: *Towards a Theory of Instruction*. Harvard University Press, 1966. 44–45.

[5] David TALL, Mercedes MCGOWER, Phil DEMAROIS: *The function machine as a cognitive root for the function concept*. 2000. (Later TALL-MCGOWER-DEMAROIS) https://www.researchgate.net/publication/238344550_The_function_machine_as_a_cognitive_root_for_the_function_concept

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presented in the above chapter (theoretical background) in this paper, the researchers decided to structure the learning environment in a specific manner so that it facilitates students to create the symbolic representations of functions. To achieve this goal, the authors meticulously designed learning environments that incorporated: concrete objects, household, and do-it-yourself devices (i.e. handmade function machine) and visual aids, while role-playing facilitated discussions that helped gradually build understanding the function concept.

In doing so, learners were exposed to diverse experience and perspectives on the concept of functions. By constructing their own concrete and visual representations, students got engaged with the learning material on a deeper level. This approach fosters active participation and hands-on learning, leading to a deeper comprehension of the abstract mathematical concept of functions. Thus, the authors formulated the following research question:

Q1: Can arranging the learning environment in a certain way help students create symbolic representations of functions?

To address this question, researchers conducted three 60 minutes interventions with 16 students, who attended 6th grade in a non-standard school, a Waldorf School in Szekszárd, Hungary.

Function machine which inputs are fruits and nuts

The goal was to introduce a representation of function machine as a representation of function, and to emphasize the attribute of the function concept, which is that each element in a domain set is mapped into exactly one element of the codomain set. There were no numbers here, so instead of measurable quantities, by using a qualitative transformation process known from everyday life, which was considered to be closer to students.

During the first intervention which started with an introductory discussion, students' attention was drawn to the simple household devices around them. Questions that were asked helped students analyzing in detail the components, accessories, and principles of operations of the manual and electric machines.

In the discussions students realized that machines determine their inputs; for instance, a juicer can only take fruits and it cannot take nuts or seeds. Students also concluded that machines transform given input into only one corresponding output i.e. by squeezing lemon, lemon juice is produced, and grinding walnut results in ground walnut. These conclusions would be crucial in understanding and creating the knowledge related to functions. At the beginning of the action process the students were divided into two groups and they were given the instructions: to put together a performance about the juicing and grinding tranformation process.

One group chose to work with the juicer, the other with the grinder; so, one was the juicer team and the other the grinder team. Several items were placed on a table such as a manual grinder and juicer, various nuts, seeds, fruits (picture 1).

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Tools and supplies: grinder, juicer, fruits, seeds, nuts (picture 1.)

The teams chose the appropriate items according to their tasks. Students in the juicer team took the juicer and the fruits that could be squeezed, whereas the grinder team chose the grinder and nuts. They considered that poopy seeds should be left on the table since there was no corresponding machine.

They had to create a model of juicer and grinder using the accesories they could find in the classroom. They used tables, tablecloths, chairs, papers, glasses, boards, trays, in order to build up a device similar to the real „machine“ (juicer or grinder). As for the input and output they had the fruits and nuts and the corresponding tools so that they could prepare the output (juice and ground nut). During the compilation of the mini-play that showed the process of squeezing (or grinding) , the students collaborated not only in the process of designing the stage elements but also in asigning the roles: some of them were involved in interacting with the audience, others took on the role of helpers behind the scenes, as well as there were students who were responsible for sound effects such as machine noise.

In order to show which products their machine could produce, the students paired up and the corresponding fruits and liquids, that is how they explained which juice originated from which fruit. In other words, the inputs were placed on the righthand-side and the corresponding outputs to lefthand-side on the table, and the pairs were put one below the other. (see picture 2).

This way they got the concrete representation of the table representation of the function.

Concreate representation of a table (picture 2.)

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Some students' idea was to make posters for their performance, which would show the process. After discussion, the members of the group agreed that the most important fact about squeezing was that they got different-coloured juices, so they decided to draw the fruits and juices as they were arranged on the table. This way they got the table of the function where input and output were drawn. One member of the group considered that instead of drawing, it would be easier to write down the names of the fruits and corresponding juices.

That's how they spontaneously got the table representation of the function where the input and output elements were represented both in drawings and words. (see picture 3).

Visual representations of a table (picture 3.)

Both groups performed their plays to the students from the other group. (see picture 4).

Students roleplay (picture 4.)

Function machine which inputs are concreate representations of numbers

As a second intervention, in the next lesson, students used a do-it-yourself function machine made by researchers, for ther acti
vities. (see picture 5)

Function machine-from a machine operator perspective (picture 5.)

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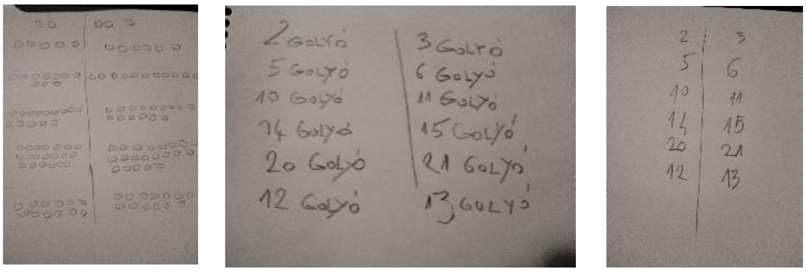
The machine worked on a principle that after inserting a certain number of balls into the input slot – according to the „working rule“ of the machine – some balls fell out of the output slot. One student played the role of the machine (machine operator), so he or she was the one, who chose the rule according to which the machine would work. And standing behind the machine, he or she carried out the rule by adding or distracting the appropriate number of balls in third slot, that was hidden from the audience. (see picture 6).

Function machine-from an audience perspective (picture 6.)

In the interactive play, the first student who played the role of the machine chose the rule which was: to add one ball to the balls that were dropped in the input slot. When one of the members of the "audience" put two balls in the input slot, the student who played the role of the machine, inserted one more ball, so the audience saw three balls falling out from the exit slot. When the other member of the audience put five balls in the input slot, the machine "dropped out" six balls etc. In order to help the "audience" solve the task, which was to find out the principle of the operation of the machine, students were urged to make posters where they took notes about each transaction. Some students used visual representations (circles), some used descriptive way of notes (using numbers and words), and some student used numbers, as symbolic representations for balls. (see picture 7).



Representations (picture 7.)

After making the posters students were asked to show them. The student who played the role of the machine asked the audience to figure out the rule by which the machine worked. Students answered that the machine added one ball to the input balls.

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Then they were asked to point out which one among the posters was the simplest to describe what the machine did. They chose the one where numbers were used only. Then students were asked to copy the poster on the blackboard, and to write it more general, namely the description of the input elements. They added "szám" (number) bellow the last number in the first column, and "az adott számnál eggyel nagyobb szám" (the number that is one greater than the given number) bellow the last number of the output elements (see picture 9.).

Dependent and independent variable in the table (picture 9.)

They were also suggested to indicate the transformation in another way, since mathematics is universal language, and we could not use Hungarian words. They agreed, to used only the first letter, so they wrote " sz" and "a" in the next row of the table. After we agreed, to use only one letters, so they put "s" and "a" in a next row. The next step towards generalization of the symbolic representation was that they figured out which letters would they use, if we translate the adequate words into different languages. They found out, that the letter "n" could stand for "s" if they use the adequate English word, namely number, and letter 't' instead of "a". They went further, so the put "z" (zahle), and "d" from adequate German words.

Next, they had to come up with an idea how they would interpret the working mechanism of the machine if they only saw this poster and had not ever seen the machine at all. After a short discussion they agreed that the imaginary machine added one to the input number and showed the output number, which was one more than the input number. In order, to emphasize what the imaginary machine did, one student added on the right-hand side of each output the notation of output which highlighted the adding of number one, so the third column was created. (see picture 10.)

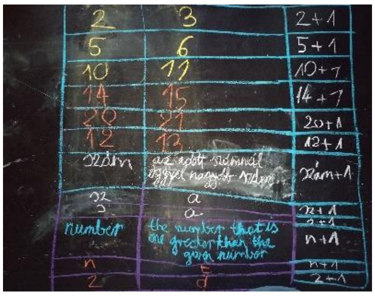

Table representation of the function y=x+1 (picture 10.)

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Then students, as they considered, that the second and the third column are equivalent, drew another table, were they by indicating this fact named the table. (see picture 11.)

On the next class several students played the role of the machine. Each figured out a different rule for the machine, in that way functions such as: y=x+1; y=2x; y=x+2; y=const., y=rand. were represented.

Than, children were given several table representations of some functions, like the examples from didactic book for teaching mathematics in lower grade of primary schools.[6] (picture 11) which they solved successfully.

I	III
II	IIII
III	
IIIII	
	IIIII
	IIIIII

s	a=s+1
5	
	10
32	
2	
	12

n	k=2*n
1	
3	
	8

(picture 11.)

Conclusion

In this study we sought the response for the question: could we by arranging the learning environment in a certain way help students to create symbolic representations of functions?

Researchers addressed students a non-standard approach to the introduction of functions, which meant that contrary to standard learning style in Hungarian schools – where students use only visual representations of function machines – here students first dealt with concrete representations instead of the visual ones. During the interventions it was proved that the diverse tools and environment can support the process of creating the notion of functions on their own.

In addition, when students were engaged in group activities like role-playing games, they had the chance to discuss and interact socially, which further enriched their understanding and contributed to the development of the function concept.

[6] Tiborné PETZ: *Függvények tanítása*. In: Eszter KÓNYA HERENDINÉ (ed.): *A matematika tanítása az also tagozaton,* Nemzedékek Tudása Tankönykiadó, Bp., 2013. 333–336.

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Absztrakt

A SZEREPJÁTÉKOKBAN REJLŐ LEHETŐSÉGEK, MELYEK SORÁN KONKRÉT ÉS VIZUÁLIS REPREZENTÁCIÓKAT HASZÁLUNK A FÜGGVÉNY FOGALMÁNAK TANÍTÁSÁRA

Ez a tanulmány bemutat egy kísérleti kutatást a függvény fogalmának tanításáról és tanulásáról egy nem hagyományos – Waldorf – iskolában. A hatodik osztályos diákok szerepjáték segítségével bővítették a függvény fogalmának elsajátításához szükséges tudásukat úgy, hogy használták a függvénygépek konkrét, és vizuális reprezentációit. A kutatók célja az ilyenfajta tanítás alkalmazásával az volt, hogy egy olyan gazdag tanulási környezet teremtsenek meg a tanulók számára, amely lehetővé teszi, hogy a diákok felfedezhessenek dolgokat és különböző tapasztalatokat szerezhessenek, és ezek segítségével megalkossák a lineáris függvények egyenleteit.

Kulcsszavak: matematika, függvények, konkrét reprezentációk, vizuális reprezentációk, szerepjáték